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| Name: | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ | Date: *\_\_\_\_\_\_\_\_\_\_\_* | |
|  | **Mathematics Specialist Unit 2**  **Investigation 3- 2018**  **Topic – Matrices**  **Take home component** |  | |
| **Date out:** | *Week \_\_\_\_ Date \_\_\_\_\_\_\_\_\_\_* | |  |
| **Take home component weighting:** | *0% of the year* | |
| **In-class component weighting:** | *5% of the year.* | |
| **Task conditions**  The students will have to complete the preparation activity and then will be allocated 30 minutes for the in-class validation, where the skills used will be assessed. Students will not be expected to remember the rules used in the Take home component part but should be able to apply them.  **Course-related information**  The concepts and skills included in this investigation relate to the following dot points within the WA Mathematics Specialist syllabus:  2.2.2 define and use addition and subtraction of matrices, scalar multiplication, matrix multiplication, multiplicative identity, and inverse  2.2.3 calculate the determinant and inverse of 2 × 2 matrices and solve matrix equations of the form  AX = B, where *A* is a 2 × 2 matrix and *X* and *B* are column vectors  2.2.8 define and use inverses of linear transformations and the relationship with the matrix inverse | | | |

**Background information**

Students should be able to do basic calculations with matrices, including working with inverses and determinants and be able to use their calculator to perform matrix operations. Students should also be able to do proofs by mathematical induction and to calculate using surds. Students should be familiar with the Fibonacci sequence.

**Fibonacci using matrices**

The Fibonacci sequence is the set of numbers 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...

The Fibonacci sequence is defined as *Fn+2 = Fn + Fn+1* with. *F1* = 1 and *F2 =* 1 where *Fn* stands for the *n*th Fibonacci number.

**Question 1**

It has been conjectured that 

Test the rule for *n* = 1, 2 and 3, for example for *n =* 1, 

**Question 2**

It has also been conjectured that 

(a) Test the conjecture for *n* = 2, 3 and 8.

(b) Assume the conjecture is true for *n* = “*n*” and test the validity for *n* = “*n +1*”.

Write down your conclusion.

(c) Given  then it follows that .

Calculate the determinant of each side of the equation and hence prove that .

(d) Test the rule  for *n* = 2, 3 and 4.

(e) (i) Form a sequence of matrices 

in simplified version, giving the first six terms of the sequence.

(ii) Use your sequence to write down a simplified matrix for.

**Question 3**

(a) Use the matrix method determined in Question 2 to identify F30.

Explain your method clearly.

(b) With your matrix method to identify F30 which other two Fibonacci numbers can be determined?

**Question 4**

Let *M* = then *M n* = 

Using a method called diagonalisation of matrices....

(a) Given *S* = . State the inverse of *S* (= *S-1*) in terms of *a, b, c* and *d*.

(b) Let *D* =  and *M = S* × *D* × *S-1*

Then *M 2 = S* × *D* × *S-1* × *S* × *D* × *S-1 = S* × *D2*× *S-1*

*M 3 = S* × *D* × *S-1* × *S* × *D2*× *S-1= S* × *D3*× *S-1*

Provide an expression for *M n* in terms of *S, D* and *S -1*

(c) Given *D* = , identify expressions for *D2, D3* and *Dn*. Explain your method.

(d) It is known that for **this** matrix, *M* =, the diagonal matrix *D*,

*D* =  and *S* = .

Determine the matrix *S* -1, expressing the elements of the matrix in exact values.

(e) Consider *D* = 

then *Dn* = 

so *M n = S* × *Dn* × *S-1* becomes

*M n* = ××

Show that this expression can be simplified to

*M n* = 

(f) Given use the expressions found in (e) to write down an expressions for *Fn* in terms of .

(g) Determine *F1, F2* and *F3* using the expressions found in (f).

**FOOTNOTE**

By solving the equation *x2 – x –* 1 *=* 0, then the solution is  .

is called the golden ratio.

\*\* Extension: Investigate the golden ratio.

**Part 1:** **Preparation activity**

**Solutions**

**Question 1**

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| Solution |

**Question 2 (a)**

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| Solution  (a) |

**Question 2 (b)**

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| Solution  Assume  Test for *n* = “*n+*1” |

**Question 2 (c)**

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| Solution |

**Question 2 (d)**

|  |
| --- |
| Solution |

**Question 2 (e)**

|  |
| --- |
| Solution  i =    ii |

**Question 3 (a)**

|  |
| --- |
| Solution  Since |

**Question 3 (b)**

|  |
| --- |
| Solution |

**Question 4 (a)**

|  |
| --- |
| Solution  *S-1* = |

**Question 4 (b)**

|  |
| --- |
| Solution  *M n = S× D n × S -1* |

**Question 4 (c)**

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| --- |
| Solution    *…* |

**Question 4 (d)**

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| Solution  *S-1=* |